LEFT DISTRIBUTIVITY

$$\forall\, a, b, c \in \mathbb{N}, \quad a(b + c) = ab + ac$$

Fix a and b. Prove the statement by induction on c.

BASE CASE c = 0

$$a(b + 0) = ab = ab + a \cdot 0$$ recursive definitions

INDUCTIVE STEP

Assume $a(b + c) = ab + ac$ induction hypothesis

$$\begin{aligned} a(b + S(c)) &= aS(b + c) && \text{addition} \\ &= a(b + c) + a && \text{multiplication} \\ &= (ab + ac) + a && \text{induction hypothesis} \\ &= ab + (ac + a) = ab + aS(c) && \text{associativity} \cdot \text{multiplication} \end{aligned}$$

$$\therefore \quad a(b + c) = ab + ac$$